Modeling and Control of an Omni-Directional Vehicle with Mecanum Wheels

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Abstract—In this work a simplified dynamic model for an omnidirectional vehicle, equipped with four Mecanum type wheels, is proposed. We admit a functional interdependence between rollers slippage and friction parameters. It is shown that by using an accurate model for slippage, higher vehicle traction performances could be achieved. Two vehicle control strategies are simulated and their performance described.

I. INTRODUCTION

The wheel slip problem with one degree of freedom is treated in many publications (for example [1] and [2]). For tractive forces modeling, a functional dependence between ground friction coefficient and wheel slip is used. The above studies can be easily applied for platforms with two degrees of freedom of the wheels, where slip influences are bigger because of mobile parts free rolling. Usually omnidirectional wheeled vehicles modeling is made without considering slippage. As a result, for small movement speeds of the platforms is often made using only the kinematic vehicle model [4]. The use of linear dynamic model is justified in a high adhesion coefficients and smaller acceleration case. To keep the control in low adhesion or forced maneuver conditions, slip inclusion is absolutely necessary. This paper is presenting such a model for omnidirectional vehicle equipped with mecanum type wheels. In the final section several strategies for vehicle control are proposed.

II. THE MODEL

A. Adhesion coefficient

To define the variation of the adhesion coefficient is necessary to specify the slip quantitatively.

\[ l_i = \frac{(\omega_i - \omega_i^*)}{\max(|\omega_i|, |\omega_i^*|)} \]  (1)

Where \( \omega_i \) is the angular speed of the wheel \( i = 1, 4 \), \( \omega_i^* \) the transfer matrix in inverted kinematic model of the vehicle, \( V_x \), \( V_y \) and \( \Omega \) are linear and angular speeds of the vehicle. This way defined we consider a functional dependence between friction coefficient with the ground.

\[ \mu_i = \mu_i(l_i) \]  (3)

Where \( \mu_i \) is the adhesion coefficient of wheel \( i \). A typical dependence is shown in Fig.1.

Fig.1 A typical adhesion coefficient dependence of wheel slip
From mentioned above the following relation results:

\[ F_t = \mu(l) N \]  

(4)

Where \( N \) is reaction force of the support and \( F_t \) is a tractive force.

B. Kinematic model

In this paper we consider a planar vehicle with four mecanum wheels as shown in Fig. 2. The rollers are disposed at 45 degree angle to the wheel axis. Inverse kinematic model deduction was treated in [4]. Inverse kinematic model is given in [5].

\[
\begin{pmatrix}
\omega_1' \\
\omega_2' \\
\omega_3' \\
\omega_4'
\end{pmatrix} =
\begin{pmatrix}
-1 & 1 & -(l_1 + l_2) \\
1 & 1 & l_1 + l_2 \\
-1 & 1 & l_1 + l_2 \\
1 & 1 & -(l_1 + l_2)
\end{pmatrix}
\begin{pmatrix}
V_x \\
V_y \\
\Omega
\end{pmatrix}
\]  

(5)

C. Dynamic modeling

Fig. 4 presents the disposition of forces that influence the vehicle. Where \( F_m^i \) is the force at rollers level generated by torque applied to wheel, \( F \) is the force applied by wheel to vehicle chassis, \( F_x \) and \( F_y \), resulting forces at the center of weight, \( F_w \) applies rotationally momentum, \( F_i \) inertial force of wheel at point of ground contact.

\[
\begin{pmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 \\
\omega_4
\end{pmatrix} = 
\frac{1}{Jr} \begin{pmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{pmatrix} - \sqrt{2} R / Jr \begin{pmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4
\end{pmatrix}
\]  

(6)

Equation (6) describes wheel level forces’ equilibrium. Where \( T_i \) is torque applied to wheel \( i \), \( Jr \) is the wheel moment of inertia, \( R \) – wheel radius.
\[
\begin{pmatrix}
F_x \\
F_y \\
F_w
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
-1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 \\
-S & S & S & -S
\end{pmatrix} \begin{pmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4
\end{pmatrix}
\]

Equation (7) describes the forces' equilibrium at the vehicle's chassis.

\[
S = \frac{l_1 + l_2}{\sqrt{l_1^2 + l_2^2}}
\]

In (9) \( m \) is vehicle mass and \( Jv \) is chassis moment of inertia. Combining results from (1),(4),(5) , actual wheel speeds can be calculated \( F_i; i = \{1,4\} \). Using this results and applying it to (6),(7)and (9) , a non-linear space-state model can be derived.

\[
X = \begin{pmatrix}
V_x \\
V_y \\
\Omega \\
\omega_1 \\
\omega_2 \\
\omega_3 \\
\omega_4
\end{pmatrix}
\]

\[
\frac{dX}{dt} = A\mu(X) + BT
\]

\[
B = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1/J_r & 0 & 0 & 0 \\
0 & 1/J_r & 0 & 0 \\
0 & 0 & 1/J_r & 0 \\
0 & 0 & 0 & 1/J_r
\end{pmatrix}
\]

Where \( \mu(X) \) is a function that gives the tractive forces, calculated from (1),(2),(3),(4).

III. Controlling

By “vehicle control” we understand bringing the linear and angular speed to desired values. In this way, imposing the vehicle speed we are actually imposing the wheel speed. The above state is valid only when there no slip present, but for the platform control in closed loop, this strategy is acceptable. To achieve necessary speed for each wheel, one PID controller is used. For the purpose in hand we assume that the vehicle is equipped with wheels encoders and accelerometers, used to determine the vehicle speed on each moment of time. For slip control we used an Anti-windup like technique, with difference that limitation is done to slip value and not to command. Hence the slip is calculated using sensors’ data by means of inverse kinematic model.

IV. Simulation

We use an approximation of adhesion coefficient, see Fig.5.

Fig.5 Functional adhesion coefficient, approximation
The first takes advantage of knowing the slippage and limits it, and the second one does not. As input signals we chose a sequence of pulses as is shown in Fig.6.

Changes in adhesion are simulated by varying $\mu_{\text{max}}$ in 0.05 and 0.85 limits. Results are shown in Fig.7.

It is clear that for higher adhesion coefficients, the behavior of both systems is appropriate, but for smaller ones system with slip limitation offers much better results. As can be seen in Fig.7 bigger difference in control strategies is notable in the case of complex movements (for example, when the rotation and translation appears at the same time).

V. CONCLUSION

In this paper we have investigated effects of wheel slip in the omnidirectional platform equipped with Mecanum type wheels. We have observed that for better traction performances, especially in complex maneuvers, there is necessary to limit torque in conjunction with slippage.

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